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ELECTROSTATIC FIELDS AND THE MAXIMUM HEAT FLUX

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Abstract—A theoretical study is made of the effect of electrostatic fields upon the maximum heat flux during pool boiling on a large horizontal cylindrical heater. Use is made of the hydrodynamic approach to the maximum heat flux. Good agreement is obtained between the theoretical predictions and the experimental observations.

NOMENCLATURE

A, C, D, F, H, amplitudes of perturbations;
B, Bond number based upon the film thickness;
B_R, Bond number based upon the heater radius;
D, column diameter;
E, electric field;
G, defined by equation (19);
J, Bessel function;
K, L, P, Q, S, U, coefficients;
N, Bessel function;
R, column radius;
R_k, R_l, principal radii of curvature of the interface;
T, electrical charge density;
V, speed;
b, column separation;
$c_{m,n}$, nth root of Bessel function J_m;
d, vapor film thickness;
g, gravitational constant;
h_{fg}, heat of evaporation per unit mass;
k, wavenumber;
p, Bessel function scale factor;
q, heat flux;
t, time;
r, x, y, z, coordinates;
ΔT, temperature difference between heater and liquid;
ΔV, voltage difference;
∇, gradient;
∇^2, Laplacian.

Greek symbols

θ, hydrodynamic potential;
ϕ, electrical potential;
ε, dielectric permittivity;
η, interfacial displacement;
λ, wavelength;
λ_d, most dangerous wavelength;
ρ, density;
ρ_e, electrical conductivity of the liquid;
σ, surface tension;
τ, charge relaxation time;
φ, cylindrical coordinate.

Superscripts

$\tilde{}$, perturbations.

Subscripts

d, most dangerous;
f, refers to the liquid;
g, refers to the gas;
m, order of the Bessel function;
max, maximum;
min, minimum;
n, order of the root of the J_m Bessel function;
0, steady-state quantity;
x, y, z, x, y and z components;
1, 2, refers to regions 1 and 2.

INTRODUCTION

THE EFFECT of electric fields upon the phenomena occurring during boiling has been studied by several investigators. Bochirol *et al.* [1] were the first to report increased heat-transfer rates when applying an AC voltage difference between two heated wires in a dielectric liquid. Bonjour and Verdier [2] showed these effects to be due to dielectrophoresis. Choi [3] also observed increased heat fluxes in his experiments with DC fields applied to a cylindrical heater in a dielectric. Also here most of the increase in heat transfer is due to electrophoresis. Markels and Durfee measured the effect of a DC field upon the maximum heat flux during pool boiling [4] and of an AC field during forced convection boiling [5].

In the present work the effect of an electrostatic field upon the maximum heat flux during pool boiling will be studied in order to explain the large increases in the maximum heat flux observed in experiments. In particular a flat plate heater located in a conducting liquid will be considered. The maximum heat flux will be determined for this configuration and will be compared to the data of Markels and Durfee [4]. To do this first the hydrodynamic theory predicting the maximum heat flux during ordinary pool boiling will be presented in the next section. This will be followed by a stability analysis of the vapor film covering a heater during film boiling, including the effect of a DC field. The results

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of this stability analysis will be applied to the flat plate heater under consideration and comparison between theoretically predicted and experimentally observed maximum heat fluxes will be made. After a discussion, the conclusions from all this will be drawn in the last section of the paper.

MAXIMUM HEAT FLUX CORRELATION

If the temperature difference ΔT between a heater and the heated liquid is steadily increased, the heat flux q behaves in a manner as depicted in Fig. 1. After a region of natural convection, boiling of the liquid at the heater occurs. Bubbles detach from the heater and rise through the liquid on account of their buoyancy (nucleate boiling). At increasing ΔT a point A of maximum heat flux occurs. Further increase leads through an unstable region to a point B at which the heat flux is minimal. From the point B on one is in the region of film boiling: a vapor film completely covers the heater and the heat flux is considerably reduced compared to the nucleate regime.

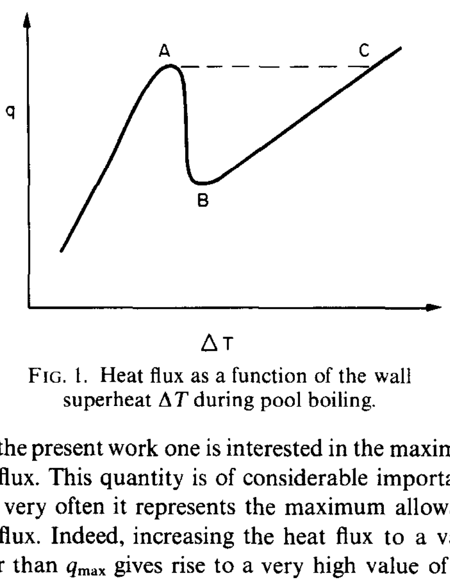

FIG. 1. Heat flux as a function of the wall superheat ΔT during pool boiling.

In the present work one is interested in the maximum heat flux. This quantity is of considerable importance since very often it represents the maximum allowable heat flux. Indeed, increasing the heat flux to a value larger than q_{max} gives rise to a very high value of ΔT (point C) which may destroy the heater.

The most successful approach to the prediction of the maximum heat flux is provided by the so-called "hydrodynamic theory". This is based upon the observation that the heat transported from the heater is contained in the heat of evaporation of the vapor. Furthermore it is observed that upon approaching q_{max} the vapor bubbles coalesce into vapor columns. These columns are spaced regularly a distance b apart (Fig. 2) and their diameter D is about $b/2$ (Zuber [6]). The maximum heat flux can then be approximated by means of the expression:

$$q_{max} = \rho_g h_{fg} \frac{V_g \pi \left[\frac{b}{4}\right]^2}{b^2} = \frac{\pi}{16} \rho_g h_{fg} V_g \qquad (1)$$

in which V_g is the vapor speed in the columns at the occurrence of q_{max}.

The hydrodynamic approach then consists in finding a suitable value for V_g. It is known that when two

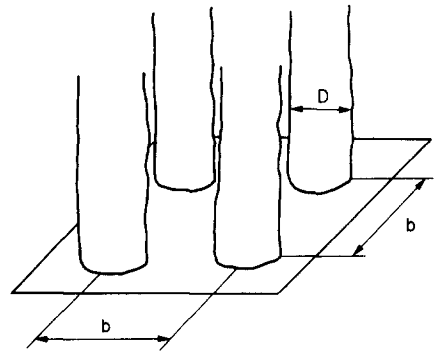

FIG. 2. Vapor column configuration on a flat plate heater.

immiscible fluids are in relative motion, their contact surface is unstable under the action of inertia and surface tension forces (Helmholtz instability). In particular in the case of a gas flowing over a stagnant liquid with the speed V_g, and for a horizontal interface between them, there exist unstable perturbations of the interface which grow exponentially in time. The wavelength of these unstable perturbations is larger than the wavelength λ related to the gas speed by (see Lamb [7]):

$$V_g = \left[\frac{2\pi\sigma}{\rho_g \lambda}\right]^{\frac{1}{2}}. \qquad (2)$$

Also the vapor–liquid interface of a vapor column is unstable. This leads to the well-known break-up of vapor columns in separate bubbles. Although the geometry is cylindrical here and not flat, Lienhard and Dhir [8] have shown that a good estimate of the value of V_g at which instability of the column sets in is obtained by assuming λ to be equal to the circumference of the column. The radius of the column is about a quarter of the spacing between the columns. The only parameter left to be determined therefore is the column spacing.

In the case of film boiling a vapor layer covers the heater. The vapor–liquid interface thus formed is unstable. The growth rate of the unstable perturbations depends upon their wavelength. The wavelength of the perturbation with the largest growth rate is called the most dangerous wavelength λ_d. It has been observed that the spacing between the columns formed just before the maximum heat flux is reached is equal to the most unstable wavelength corresponding to film boiling over the same heater. It has also been shown in [8] that for cylindrical heaters one should probably take:

$$\lambda = \lambda_d.$$

Substituting this into (2) yields:

$$q_{max} = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{\pi}{8} \left[\frac{\rho_g \sigma}{\lambda_d}\right]^{\frac{1}{2}} h_{fg} \qquad (3)$$

which shows that λ_d is the only quantity which remains to be determined to evaluate q_{max}.

For a flat plate heater λ_d is (see [8]):

$$\lambda_d = 2\pi 3^{\frac{1}{2}} \left[\frac{\sigma}{g(\rho_f - \rho_g)}\right]^{\frac{1}{2}}.$$

This last expression is valid only for zero electric field since the stability of the film interface also depends upon the presence of such a field. Therefore in the following section a new expression for λ_d, taking into account electric field effects, will be derived.

The effect of the electric field upon the wavelength λ which characterizes the instability of the vapor columns will be neglected here. The vapor columns collapse at a distance from the heater which is many times the film thickness during film boiling. Considering that the field strength, and thus the electrical surface force density, decrease rapidly away from the heater (see Appendix), it follows that the effect of the electric field will be primarily to change λ_d.

STABILITY ANALYSIS

In order to determine λ_d a stability analysis of the vapor film formed over a flat horizontal heater will be performed. A cartesian coordinate system (x, y, z) with its x–y plane coinciding with the vapor–liquid interface will be used here (see Fig. 3). This interface will be perturbed. Depending upon the wavelength of these perturbations, they will grow steadily in time or oscillate. The latter are stable perturbations while the former are unstable ones. The wavelength of the perturbation with the largest growth rate will be determined.

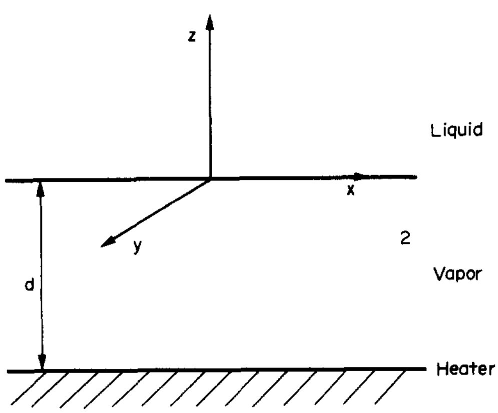

FIG. 3. Vapor film on a flat plate heater.

It will be assumed that both liquid and vapor are incompressible, inviscid, stagnant and immiscible fluids. The vapor is a perfect insulator while the liquid and the heater are assumed to be perfect conductors. A potential difference ΔV exists between these last two giving rise to the uniform electric field E in the vapor film and to the surface charge density T upon the heater surface and the vapor–liquid interface. Since field and charge density give rise to the force density $TE/2$ acting upon the interface, it may be expected that they will enter into the stability analysis.

It is assumed that the interface $z = 0$ is displaced in the z-direction over the amount η with:

$$\eta(x, y, t) = A\, e^{i\omega t} e^{i(k_x x + k_y y)}. \tag{4}$$

This displacement gives rise to perturbations in the pressure and velocity fields. They can be written in the form:

$$p_1(x, y, z, t) = p_{0,1}(x, y, z) + \tilde{p}_1(x, y, z, t),$$
$$p_2(x, y, z, t) = p_{0,2}(x, y, z) + \tilde{p}_2(x, y, z, t),$$
$$\mathbf{v}_1(x, y, z, t) = 0 + \tilde{\mathbf{v}}_1(x, y, z, t),$$
$$\mathbf{v}_2(x, y, z, t) = 0 + \tilde{\mathbf{v}}_2(x, y, z, t).$$

In the analysis it will be assumed that the perturbations are small compared to the zero-order quantities (linearized analysis). Considering only terms of first order in the perturbations, the momentum equations of the two fluids yield:

$$\rho_1 \frac{\partial \tilde{\mathbf{v}}_1}{\partial t} = -\nabla \tilde{p}_1,\qquad \rho_2 \frac{\partial \tilde{\mathbf{v}}_2}{\partial t} = -\nabla \tilde{p}_2. \tag{5}$$

Here use is made of the fact that the two fluids are stagnant in the steady-state.

These equations, together with the fact that:

$$\frac{\partial \tilde{\mathbf{v}}}{\partial t} = i\omega \tilde{\mathbf{v}}$$

show that the perturbed velocity fields can be derived from velocity potentials. To determine these potentials, use is made of the continuity equation:

$$\frac{\partial \tilde{v}_x}{\partial x} + \frac{\partial \tilde{v}_y}{\partial y} + \frac{\partial \tilde{v}_z}{\partial z} = 0$$

which is valid in both regions. In terms of the velocity potential $\tilde{\theta}$, this continuity equation can be written as:

$$\nabla^2 \tilde{\theta} = 0$$

since (6)

$$\tilde{\mathbf{v}} = \nabla \tilde{\theta}.$$

In these last equations the subscripts 1 and 2 have been omitted to indicate that they are valid in both regions. A solution to (6) which has the same time and $x-y$ dependence as η can be written as (see Lamb [7]):

$$\tilde{\theta} = \frac{e^{i\omega t}}{k} e^{i(k_x x + k_y y)} [D\, e^{kz} - C\, e^{-kz}]$$

with

$$k^2 = k_x^2 + k_y^2. \tag{7}$$

This gives rise to, say, a $\tilde{v}_{1z}$ field of the form:

$$\tilde{v}_{1z} = e^{i\omega t} e^{i(k_x x + k_y y)} [C\, e^{-kz} + D\, e^{kz}]. \tag{8}$$

The coefficient D in (8) has to be zero since otherwise it would give rise to infinite velocities far away from the interface, which is physically impossible since it would assume infinite kinetic energy. Similarly to the $\tilde{v}_{1z}$ solution, a $\tilde{v}_{2z}$ solution exists which is irrelevant in the present analysis as will be found later.

At the interface the kinematic condition:

$$\frac{\partial \eta}{\partial t} = \tilde{v}_{1z} = \tilde{v}_{2z} \quad \text{at} \quad z = 0 \tag{9}$$

has to be satisfied. Substituting (4) and (8) into (9) leads to the result:

$$\tilde{v}_{1z} = iA\omega\, e^{i(k_x x + k_y y)}\, e^{-kz}\, e^{i\omega t}. \tag{10}$$

Substituting this last result into (5) it is found that:

$$\tilde{p}_1 = -\rho_1 \frac{A\omega^2}{k} e^{i(k_x x + k_y y)}\, e^{-kz}\, e^{i\omega t}. \tag{11}$$

The pressure difference across the interface has to satisfy Laplace's condition for capillary phenomena

$$p_1 - p_2 = \sigma\left[\frac{1}{R_l} + \frac{1}{R_k}\right] - \frac{TE}{2}$$

in which the electric force density is taken into account. The charge density T at the surface of a conductor is related to the electric field at the conductor by (see Landau and Lifschitz [9]):

$$T = \varepsilon_g E.$$

The above force condition at the interface therefore becomes:

$$p_1 - p_2 = \sigma\left[\frac{1}{R_l} + \frac{1}{R_k}\right] - \frac{\varepsilon_g E^2}{2}. \tag{12}$$

It should be noted that the electric force is independent of the direction of E: it is always directed in a direction away from the liquid and into the vapor. Equation (12) has to be satisfied at all times.

The pressure at the perturbed interface can be written as:

$$p_1(x, y, \eta, t) = p_{0,1}(x, y, 0) + \left.\frac{\partial p_{0,1}}{\partial z}\right|_{z=0} \eta(x, y, t) + \tilde{p}_1(x, y, 0, t)$$

$$p_2(x, y, \eta, t) = p_{0,2}(x, y, 0) + \left.\frac{\partial p_{0,2}}{\partial z}\right|_{z=0} \eta(x, y, t) + \tilde{p}_2(x, y, 0, t).$$

The variation of the mean curvature of the interface due to the displacement η in the z-direction, considering only terms of first order in η, is given by:

$$\widetilde{\left[\frac{1}{R_l} + \frac{1}{R_k}\right]} = \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} = -k^2 \eta.$$

The variation of the electrical force density can be written as:

$$\widetilde{\left[\frac{\varepsilon_g E^2}{2}\right]} = \frac{\varepsilon_g}{2} \eta \frac{\partial (E_0^2)}{\partial z} + \varepsilon_g E_0 \tilde{E}_z.$$

The first term on the R.H.S. accounts for the variation of E_0 away from the interface. However since E_0 is independent of z (the field between two flat plates is uniform) the first term is zero. The second one represents the contribution $\tilde{E}$ of the change of the field configuration due to the change in shape of the interface.

Substituting these last four expressions into (12) and keeping only terms of first order in the perturbation yields:

$$\left[\frac{\partial p_{0,1}}{\partial z} - \frac{\partial p_{0,2}}{\partial z}\right]_{z=0} \eta + \tilde{p}_1 - \tilde{p}_2 = -k^2 \sigma \eta - \varepsilon_g E_0 \tilde{E}_z. \tag{13}$$

This can be further simplified by considering that in the present problem $\rho_2 \ll \rho_1$. Therefore from the momentum equations of zero order and of first order in the perturbations, it follows that:

$$\frac{\partial p_{0,2}}{\partial z} \ll \frac{\partial p_{0,1}}{\partial z}$$

and

$$\tilde{p}_2 \ll \tilde{p}_1.$$

Therefore the terms with p_2 will be neglected in (13). This explains why the $\tilde{\mathbf{v}}_2$ field is irrelevant in this problem.

In the steady-state the liquid is stagnant, therefore:

$$\frac{\partial p_{0,1}}{\partial z} = -\rho_1 \boldsymbol{h}. \tag{14}$$

Evaluating $\tilde{p}_1$ from (11) at $z = 0$ and substituting it in (13) together with (14) yields:

$$-\rho_1 g \eta - \frac{\rho_1 \omega^2}{k} \eta = -k^2 \sigma \eta - \varepsilon_g E_0 \tilde{E}_z. \tag{15}$$

In this last expression the relation between $\tilde{E}_z$ and η has to be determined.

The perturbed electric field can be derived from a potential $\tilde{\phi}(x, y, z, t)$ which in the vapor region has to satisfy the equation:

$$\nabla^2 \tilde{\phi} = 0$$

which results from the fact that the field is divergenceless (see [9]). A solution of this equation which has the same x, y and t dependence as η is:

$$\tilde{\phi} = e^{i(k_x x + k_y y)}\, e^{i\omega t}(F\, e^{kz} + H\, e^{-kz}).$$

The boundary conditions necessary to determine F and H are:

$$\eta \frac{\partial \phi_0}{\partial z} + \tilde{\phi} = 0 \quad \text{at} \quad z = 0 \quad \text{and}$$

$$\tilde{\phi} = 0 \quad \text{at} \quad z = -d.$$

The first one expresses the fact that the liquid is a conductor which is kept at constant potential while being deformed. The second one expresses the same thing for the rigid heater. This gives as solution:

$$\tilde{\phi} = -\eta E_0\left[\frac{e^{kz}}{1 - e^{-2kd}} + \frac{e^{-kz}}{1 - e^{2kd}}\right].$$

Considering that $\tilde{E}_z = \partial\tilde{\phi}/\partial z$ and $E_0 = \partial \phi_0/\partial z$ it is found that:

$$\tilde{E}_z = -\eta E_0 k \coth kd \quad \text{at} \quad z = 0.$$

Finally substituting this last expression into (15) yields the dispersion relation:

$$\omega^2 = \frac{\sigma}{\rho_1} k^3 - gk - \frac{\varepsilon_g E_0^2}{\rho_1} k^2 \coth kd. \tag{16}$$

Depending upon the value of k and the physical parameters of the system, positive or negative values of ω^2 can be obtained from this equation. Positive solutions of ω^2 give rise to pure oscillations and correspond to a stable situation. Negative values of ω^2

signify exponential growth of the perturbation and thus instability.

As can be seen from (16) the effect of surface tension is stabilizing while the effects of gravity and of the electric field are destabilizing. Since wavelength and wavenumber are related by:

$$k = \frac{2\pi}{\lambda},$$

it also follows from (16) that large wavelengths (small k) tend to be unstable ($\omega^2 < 0$), while small wavelengths (large k) tend to be stable ($\omega^2 > 0$).

In dimensionless form the dispersion relation can be written as:

$$\frac{\rho_1 \omega^2 d^3}{\sigma} = k^3 d^3 - B^2 kd - G^2 k^2 d^2 \coth kd \qquad (17)$$

in which the Bond number B is defined as:

$$B^2 = \frac{\rho_1 g d^2}{\sigma} \qquad (18)$$

and G is given by:

$$G^2 = \frac{\varepsilon_g E_0^2 d}{\sigma}. \qquad (19)$$

B^2 represents the ratio of the inertia to the surface tension forces in the system, while G^2 is the ratio of the electric forces to the surface tension forces.

Furthermore for $kd \gg 1$ the dispersion relation can be approximated by:

$$\frac{\rho_1 \omega^2 d^3}{\sigma} = k^3 d^3 - G^2 k^2 d^2 - B^2 kd. \qquad (20)$$

The wavenumber of the wave with maximum growth rate can be determined by equating the derivative of ω^2, with respect to kd, equal to zero, yielding:

$$k_d d = \frac{G^2 + (G^4 + 3B^2)^{\frac{1}{2}}}{3}. \qquad (21)$$

The solution with the minus sign for the square root term is not retained since it would give rise to negative values of k_d.

For $k_x = k_y$ the most dangerous wavenumber in the x or y directions is $k_d/2^{\frac{1}{2}}$. The most dangerous wavelength λ_d in the x or y directions is then given by:

$$\lambda_d = \frac{6\pi 2^{\frac{1}{2}} d}{G^2 + (G^4 + 3B^2)^{\frac{1}{2}}}. \qquad (22)$$

This expression will be utilized to determine q_{max}.

MAXIMUM HEAT FLUX PREDICTION

From (22) and (3) it follows that the maximum heat flux during pool boiling from a flat horizontal heater which is at a different potential than the liquid is given by:

$$q_{max} = \rho_g^{\frac{1}{2}} h_{fg} \left(\frac{\pi}{2}\right) \frac{1}{8\, 3^{\frac{1}{4}}} (\sigma \rho_f g)^{\frac{1}{4}} \left\{ \frac{G^2}{3^{\frac{1}{2}} B} + \left[\frac{G^4}{3B^2} + 1 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}}. \qquad (23)$$

For large values of G^2/B this reduces to:

$$q_{max} = \frac{1}{8}\left(\frac{\pi}{3}\right)(\varepsilon_g \rho_g)^{\frac{1}{2}} \frac{\Delta V}{d} h_{fg} \qquad (24)$$

in which the steady-state electric field E_0 is replaced by $\Delta V/d$. In these last two expressions use is made of the fact that the hydrodynamic theory gives the best results when instead of λ_d as given by (22), $\lambda_d/2^{\frac{1}{2}}$ is substituted (see Sernas *et al.* [10]).

Thus the maximum heat flux during pool boiling in the case where gravitational effects are negligible is predicted to vary linearly with the electric field ($\Delta V/d$) between heater and liquid and with the square root of the electric permittivity and the density of the vapor.

To compare this theoretical prediction of q_{max} with observations, reference will be made to the experiments of Markels and Durfee [4] with a DC electric field. A $\frac{3}{8}$ in metal tube was internally steam-heated and placed horizontally in an aluminum tank filled with isopropyl alcohol. Direct voltage was applied between the tank (high voltage) and the tube (ground). The value of the maximum heat flux as a function of this voltage difference was recorded.

In the process of deriving (23) several assumptions were made which have to be satisfied in the experiments mentioned. First it was assumed that the liquid–vapor system is stagnant, incompressible, inviscid and immiscible. As shown by Lienhard and Wong [11] the first three conditions are readily satisfied in most pool boiling experiments. Also the last condition is satisfied here since a liquid and its vapor do not mix easily. Furthermore upon applying Laplace's equation (12) at the interface it is assumed that no mass transfer occurs through the interface. Although evaporation takes place there, its effect is negligible (see Dhir and Lienhard [12]) when the experiments are performed far from the critical state. This is certainly the case in the experiments of Markels and Durfee.

Furthermore it is assumed that the liquid is a perfect conductor and that the vapor is a perfect insulator. The latter condition is certainly satisfied, as pointed out by Markels and Durfee [4]. Whether a liquid can be considered a good conductor or not depends upon the time scale involved in changes of charge density occurring in the liquid. The time scale τ in which responses to a sudden change in charge density occur (say at the vapor–liquid interface) is given by (see Moore [13]):

$$\tau = \varepsilon_f \rho_e.$$

For isopropyl alcohol: $\varepsilon_f = 18 \times 8.85\ 10^{-12} (F/m)$ and $\rho_e = 1.9\ 10^4\ \Omega m$, such that:

$$\tau = 3\ 10^{-6}\,\text{s}.$$

Thus it is found that the charge relaxation time is much smaller than the growth times of the waves considered which are of the order of 10^{-3} s. Therefore the liquid may be considered a perfect conductor: the charge density at the interface has sufficient time to adapt itself to a perturbed shape of the interface.

Estimating G (based upon $d = 1$ mm and $\Delta V = 4000$ V) leads to values of G of 2.5 and thus to values of λ_d of 2 mm, by means of (22). This means that λ_d is much smaller than the radius of the heater. Therefore many columns spaced λ_d apart occur at the heater, and therefore the flat plate limit is valid. This decreasing of the spacing between the vapor columns due to the electric field is clearly illustrated in the photographs of Markels and Durfee [4].

Not only for large values of G (and thus ΔV) but also for small values of G the flat plate limit remains meaningful. Instead of the Bond number defined upon the film thickness d one can also define a Bond number B_R based upon the cylinder radius R. In the present experiment this leads to a value of about 3 for B_R. The work of Sun and Lienhard [14] on large cylindrical heaters and of Lienhard and Dhir [8] on flat plate heaters show that q_{max} for a flat plate (as given by (23) for $G = 0$) is about 1.267 times the value of q_{max} for large cylindrical heaters. Therefore the G_{max} values given by (23) have to be divided by this factor. Thus also for small values of G (i.e. $\Delta V < 2000$ V in the present experiments) the flat plate limit remains a good approximation because of the large value of B_R.

Although for small values of G, the value of $k_d d$ is not large enough to allow the coth term to be taken equal to unity in (17), it should be noted that this term is multiplied by G^2 and thus the whole term becomes negligible.

It seems therefore that all the conditions underlying (23) are satisfied by the experiments such that $q_{max}/1.267$ from (23) may be compared to the measured values of q_{max}. This has been done on Fig. 4. The vapor film thickness d here was taken to be equal to 1.5 mm which from the photographs of Markels and Durfee [4] shows to be a reasonable guess. Furthermore ε_g was taken to be the dielectric constant of free space, $\rho_g = 1 (kg/m^3)$ and $h_{fg} = 7.26\ 10^5 (J/kg)$.

From Fig. 4 it can be seen that theory and experiment are in rather good agreement. They both show a linear increase of q_{max} with ΔV for large ΔV. The agreement between theory and experiment is not perfect however. This may be due to the choice of the vapor film thickness. Certainly this distance varies with the heat flux and does not remain constant as was assumed here.

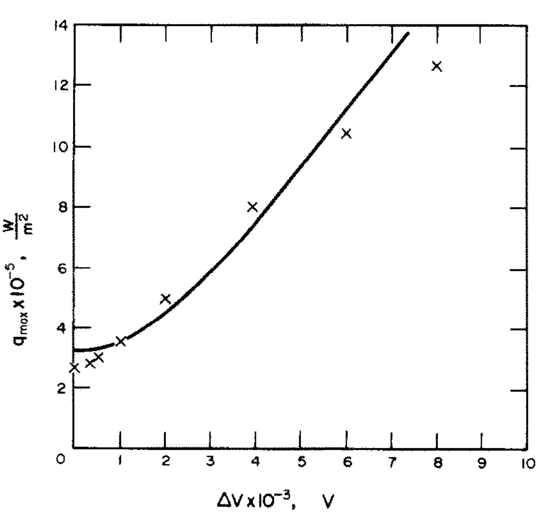

FIG. 4. Comparison of observed and calculated values of the maximum heat flux. ——, q_{max} as given by equation (23); ×, observed values of q_{max} of Markels and Durfee [4].

In particular it increases with q_{max}. This gives rise to smaller values of q_{max} at large values of ΔV than would be obtained with a constant value of d. At small values of ΔV larger values of q_{max} would be obtained than with a constant value of d. This is what is found in Fig. 4. The choice of $d = 1.5$ mm seems to be a rather good average and is acceptable: a film thickness of about 30% of the radius is not unusual.

DISCUSSION AND CONCLUSIONS

Markels and Durfee [5] in their attempt to explain their results theoretically started from an analysis of the forces acting upon a bubble detaching from a heater. Due to the complexity of this force analysis they did not arrive at estimates for q_{max}, yet were able only to determine regions in which electrostatic forces or electrophoretic forces are dominant. However the hydrodynamic theory which explains q_{max} so well for ordinary pool boiling is not based upon a force analysis, but upon hydrodynamic stability considerations. As can be seen from the photographs the character of the fluid flow is not basically altered by the application of a voltage difference: here too regularly spaced vapor columns occur. It may be expected therefore that also here the hydrodynamic approach will lead to accurate predictions of q_{max}. This explains the good results of the present analysis.

It may be concluded therefore that also when applying a DC electric field between heater and heated liquid, the occurrence of the maximum heat flux is determined by the instability of the vapor columns detaching from the heater surface. When gravitational effects can be neglected, it is found that the maximum heat flux is proportional to the applied electric field in the vapor region. This is in good agreement with experiment.

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APPENDIX

The variation of the electric field in a vapor column can be easily determined. A model consisting of a circular cylinder closed at the bottom and extending to infinity at the top will be considered. The bottom is at a potential ΔV while the sides of the cylinder are at zero potential. This model corresponds well to the case of a vapor column in a perfectly conducting liquid originating on a heater at a different potential, especially when the film thickness is small with respect to the column radius. The electrical potential describing the field in the cylinder has to satisfy the Laplace equation. In terms of the usual cylindrical coordinates a solution of Laplace's equation in the cylinder can be written as (see Jackson [15]):

$$[KJ_m(pr)+LN_m(pr)][P\sin m\varphi+Q\cos m\varphi][S\,e^{pz}+U\,e^{-pz}].$$

A general solution for the electrical potential ϕ_0 inside the column is made up of a sum of functions of this type. However since the potential remains finite at infinite z (the potential of the cylinder remains zero there), the coefficient S has to be zero. Furthermore since the potential has to remain finite at the z-axis, the coefficient L must be zero. The potential has to be zero at $r = R$. This shows that p can take only those values which satisfy:

$$p_{m,n} = \frac{c_{m,n}}{R} \qquad n = 1,2,3\ldots.$$

The electrical potential can therefore be written as:

$$\phi_0(r,\varphi,z) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} J_m(p_{m,n}r) \times e^{-p_{m,n}z}[P_{m,n}\sin m\varphi + Q_{m,n}\cos m\varphi].$$

The unknown coefficients $P_{m,n}$ and $Q_{m,n}$ can be determined from the condition that $\phi_0 = \Delta V$ at $z = 0$. It is not necessary to do this however to see how the electric field in the cylinder varies with the z-coordinate. The electric field in the direction normal to the cylinder (r-direction) at the cylinder ($r = R$) is given by:

$$\left.\frac{\partial\phi_0}{\partial r}\right|_{r=R} = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\left.\frac{dJ_m}{dr}\right|_{r=R} \times e^{-p_{m,n}z}[P_{m,n}\sin m\varphi + Q_{m,n}\cos m\varphi].$$

From this it can be seen that the electric field consists of a sum of terms which decrease exponentially with z. The smallest value of $p_{m,n}$ is $p_{0,1} = 2.4$. This means that the electric field reduces by a factor of 10 over a distance of about one column radius. The electric force density therefore reduces by a factor of 100 over one column radius. The effect of the potential difference thus remains restricted to the column region close to the heater and does not affect the stability of the vapor column at larger values of z (which remains determined by the Helmholtz instability).

CHAMPS ELECTROSTATIQUES ET FLUX THERMIQUE MAXIMAL

Résumé—On effectue une étude théorique de l'effet de champs électrostatiques sur le flux thermique maximal lors de l'ébullition à surface libre sur un grand cylindre horizontal de chauffage. On utilise l'approche hydrodynamique du flux de chaleur maximal. Un bon accord a été obtenu entre les prévisions numériques et les observations expérimentales.

ELEKTROSTATISCHE FELDER UND MAXIMALER WÄRMESTROM

Zusammenfassung—Theoretisch wurde der Einfluß elektrostatischer Felder auf den maximalen Wärmestrom beim Behältersieden an einem großen, waagerechten, zylindrischen Heizkörper untersucht. Es wurden hydrodynamische Überlegungen zugrundegelegt, und es ergab sich gute Übereinstimmung zwischen den theoretischen Berechnungen und den experimentellen Ergebnissen.

ЭЛЕКТРОСТАТИЧЕСКИЕ ПОЛЯ И МАКСИМАЛЬНЫЙ ТЕПЛОВОЙ ПОТОК

Аннотация — Проведено теоретическое исследование влияния экектростатических полей на максимальный тепловой поток при кипении в больмом горизонтальном цилиндрическом нагревателе. Максимальный тепловой поток рассматривается с гидродинамической точки зрения. Получено хорошее соответствие между теоретическими расчётами и экспериментальными данными.